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The Breakup of Trailing-Line Vortices

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It is now known that Batchelor's trailing-line vortex is extremely unstable to small amplitude disturbances for swirl numbers in the neighborhood of .83. We present results of numerical calculations that show the response of the vortex in this range of swirl numbers to finite amplitude, temporal, helical disturbances. Phenomena observed include: 1) ejection of axial vorticity and momentum from the core resulting in the creation of secondary, separate vortices; 2) a great intensification of core axial vorticity and a weakening of core momentum; 3) the production of azimuthal vorticity in the form of a tightly wrapped spiral wave. The second phenomenon eventually stabilizes the vortex, which then smooths and gradually returns to an axisymmetric state. The calculations are mixed spectral-finite-difference, fourth-order accurate, and have been carried out at Reynolds numbers of 1000-2000. Some linearized results will also be discussed in an attempt to explain the process of vortex intensification.

Applications

- Axisymmetric evolution of vortices, jets, and wakes.
- 3D instability of axisymmetric flows (weakly nonlinear).
- Spiral and axisymmetric vortex breakdown - strongly nonlinear instabilities.
- Acoustic excitation of swirling jets.
- Mixing caused by vortices - effects on combustion.

Numerical Methods

- 3D direct numerical simulation using primitive variables.
- Unbounded cylindrical coordinates - use of mapping $r = \tan\lambda$.
- Mixed spectral and finite difference methods

  Azimuthal: spectral ($\exp in\theta$).

  Axial: spectral ($\exp i\alpha m x$) or 2nd order FD.

  Radial: spectral ($\sin^n 2\lambda \ \cos 2j\lambda$) or 4th order FD.
- Fourth order accurate (Runge-Kutta) in time.

Batchelor's Trailing-Line Vortices

- Defined by axial velocity $= \exp(-r^2)$, axial vorticity $= q \exp(-r^2)$.
- Model for aircraft trailing line vortices, created in laboratory.
- Very unstable in some ranges of q. $q \simeq .85$ most unstable.
- From inviscid theory, all azimuthal wavenumbers unstable.

  As $n \to \infty$ $c_i \to .4$.

  As $n \to \infty$ most unstable axial wavenumber $\to .52n$
- From viscous theory, c_i modified by $O(R^{-1})$. # unstable waves $O(R^{3/5})$.

Numerical Solution

- Quasi-3D calculation can capture all the most unstable modes.
- Navier Stokes equations have helical solutions:

$$\sum_{n=-\infty}^{n=+\infty} f_n(r,t) \exp in(\beta x - \theta)$$

- Calculations have $q = .82$, $\beta = .52$, $R = 1600$.
- Calculations spectral in x and θ, fourth order in λ.

Features of Solution

- Initially, elliptical deformation of vortex. Helical displacement from center line.
- Kinematic deformation of vortex then produces spiral structure.
- Axial divergence intensifies axial vorticity, is associated with azimuthal vorticity.
- "Turbulent" patches form at vortex outer edges.
- In later stages, viscous diffusion weakens spiral structure. Turbulent patches separate from vortex. Vortex core remains intense.
- Intensified vortex returns to axisymmetric state.

Axial vorticity intensifies. Axial velocity weakens. Vortex stabilizes.